

The Structural Classification of Crystal Point Symmetries

BY J. D. H. DONNAY

Department of Geological Sciences, McGill University, 3450 University Street, Montreal, PQ, Canada H3A 2A7*

(Received 7 February 1977; accepted 17 May 1977)

The current classification of crystal point symmetries, which dates back to the 1935 edition of the *International Tables*, is readily updated on the basis of the lattice. The seven lattice point symmetries define the 'crystal systems'. First, for each lattice symmetry, the shape of the conventional cell and the resulting coordinate axes are chosen and labelled on metrical considerations. The possible oriented crystal symmetries are then all the subgroups of the lattice symmetry, down to its lowest merohedry, such as the ogdohedry 311 (subgroup of index 8) of the hexagonal lattice ($6/m\ 2/m\ 2/m$) or the tetarohedry 31 of the rhombohedral lattice ($\bar{3}\ 2/m$). The resulting 44 symmetry symbols are encountered in crystallographic practice. As to the celebrated 32 'crystal classes', which hark back to 1830, they are the only possibilities for wooden crystal models, which obey the Law of Rationality but ignore the orientation of the underlying crystal lattice. Combination of the 44 oriented crystal symmetries with the lattice-centering modes yields 96 oriented representations for the 73 symmorphic space groups.

Introduction

The purpose of this note is to present a classification of crystal point symmetries that is based on the lattice to serve the needs of the practicing crystallographer.

* Temporary address: Department of Physics, Arizona State University, Tempe, Arizona, USA.

On his way towards a crystal-structure determination, the crystallographer first determines the shape of the smallest conventional cell and the lattice point symmetry. He measures the cell edges, choosing and labelling them as coordinate axes in some arbitrary but unique way, so as to be able to record the systematic extinctions; these yield the mode of centering of the lattice and eventually the complete aspect symbol,

Table 1. *Classification of crystal symmetries*

See footnotes for details of column headings.

1	2	3	4	5	6
<i>c</i>	$4/m\ \bar{3}\ 2/m$ 43m 432 $2/m\ \bar{3}1$ 231	<i>cPIF</i>	$4/m\ \bar{3}\ 2/m$ 43m 432 $2/m\ \bar{3}1$ 231	<i>PIF</i>	$4/m\ \bar{3}\ 2/m$ 43m 432 $2/m\ \bar{3}$ 23
	5	15	5	15	5
<i>h</i>	$6\ m\ 2/m\ 2/m$ { 62m 6m2 6mm 622 $6/m\ 11$ 611 611 { $\bar{3}\ 2/m\ 1$ $\bar{3}1\ 2/m$ { 3m1 31m { 321 312 $\bar{3}11$ 311	<i>hP</i>	$6/m\ 2/m\ 2/m$ 62m 6mm 622 $6/m\ 11$ 611 611 $\bar{3}\ 2/m\ 1$ 3m1 321 $\bar{3}11$ 311	{ <i>P</i> <i>H</i> <i>P</i> <i>P</i> <i>P</i> <i>P</i> <i>P</i> { <i>P</i> <i>H</i> { <i>P</i> <i>H</i> <i>P</i> <i>P</i> <i>P</i>	$6/m\ 2/m\ 2/m$ 6m2 6mm 622 $6/m$ 6 6
	16	16	12	16	7
<i>r</i>	$3\ 2/m$ 3m 32 31 31	<i>rP [=hR]</i>	$3\ 2/m$ 3m 32 31 31	<i>rP = hR</i>	$\bar{3}\ 2/m$ 3m 32 $\bar{3}$ 3
	5	5	5	5	5

Table 1 (cont.)

<i>t</i>	$4/m\ 2/m\ 2/m$ $42m$ $4m2$ $4mm$ 422 $4/m\ 11$ 411 411 8	<i>tPI</i> 16	$4/m\ 2/m\ 2/m$ $42m$ $4mm$ 422 $4/m\ 11$ 411 411 7	PI PI CF PI PI PI PI PI 16	$4/m\ 2/m\ 2/m$ $42m$ $4mm$ 422 $4/m$ 4 4 7
<i>o</i>	$2/m\ 2/m\ 2/m$ $2mm$ $m2m$ $mm2$ 222 5	<i>oPABCIF</i> 30	$2/m\ 2/m\ 2/m$ $mm2$ 222 3	$P(CAB)IF$ $PC(AB)IF$ $P(CAB)IF$ 13	$2/m\ 2/m\ 2/m$ $mm2$ 222 3
<i>m</i>	$2/m$ m 2 3	<i>mPCAI</i> 12	$2/m$ m 2 3	$P(CAI)$ 6	$2/m$ m 2 3
<i>a</i>	$\bar{1}$ 1 2	<i>aP</i> 2	$\bar{1}$ 1 2	P 2	$\bar{1}$ 1 2
	44	96	37	73	32

Column 1: Initial of the name used to designate the shape of the conventional cell: cubic, hexagonal, rhombohedral, tetragonal, orthorhombic, monoclinic, anorthic (= triclinic).

Column 2: The seven lattice point symmetries, each one followed by its subgroups down to the lowest merohedry. Identity symbol 1 indicates a direction that is a symmetry direction for the lattice, but not for the crystal. The smallest conventional cell is chosen; its edges along lattice symmetry directions or shortest translations are labelled in an arbitrary, but unique, way. The number of groups in each system is shown in lower right corner. The derivation yields 44 different symbols.

Column 3: Lattice modes needed in each lattice. For each system, in lower right corner, number of combinations of crystal symmetry with lattice mode. Under *r* note the choice between two descriptions, *rP* and *hR*, where *hR* is not a mode of the hexagonal lattice. The total number of combinations is 96 (cf. columns 4–5).

Columns 4–5: This scheme differs from that of columns 2–3 in that only one subgroup out of two appears in five cases (four in the hexagonal and one in the tetragonal) and only one out of three (*mm2*) in the orthorhombic system. The 44 symmetry symbols of columns 2–3 are now reduced to 37, but the use of multiple cells is now necessary (note new lattice modes: *H* in *h*, *C* and *F* in *t*). Lattice modes collected between parentheses are not considered distinct, under *o* and *m*. The number of combinations is reduced to 73; they represent the 73 symmorphic space groups (*alias* 'arithmetic crystal classes').

Column 6: The traditional 32 crystal point symmetries, which do not take the cell into account. Strictly speaking they are the possible symmetries, not of crystals, but of wooden crystal-form models, whose orientation is not related to any lattice. The wooden models obey the Law of Rationality and the restriction that it imposes on the order of the rotation axes. Comparison of column 6 with columns 2–3 shows that the 32 point groups are inadequate to express the crystallographic facts.

with which several space groups can be compatible.

The classification of crystal point symmetries

It is natural, under the circumstances, to choose the shape of the smallest conventional cell, which depends on the point symmetry of the lattice, as the basis on which to define the systems. There are seven such cell shapes and seven lattice symmetries; correspondingly there are seven systems. Let us give (Table 1, column 1) the initial of the name that designates the shape of the smallest conventional cell (see Appendix, Note 1). *c, h, r, t, o, m, a*, from cubic to anorthic. To avoid conflict with the *t* of tetragonal, triclinic is called anorthic. A small letter is used in order to distinguish the shape of the cell from its centering mode, the latter being designated by a capital. In column 2 each system is

headed by the holohedry (see Appendix, Note 2), *i.e.* the crystal symmetry that is the same as the lattice symmetry. The holohedry is followed by *all* its merohedries, that is to say all the subgroups which, as crystal symmetries, require the holohedry as lattice symmetry. Under *h*, for instance, the last subgroup of $6/m\ 2/m\ 2/m$ is 311 , the hexagonal ogdohedry, whose order is $\frac{1}{3}$ that of the lattice symmetry; under *r* the last subgroup of $\bar{3}\ 2/m$ is 31 , the rhombohedral tetartohedry, whose order is $\frac{1}{4}$ that of its holohedry.

The number of items (three, two or one) in the symbol of a merohedry (of a lattice symmetry) – that is, a crystal symmetry within the corresponding system – is the same as the number of kinds of symmetry directions in the lattice. These directions are the *Blickrichtungen* (see Appendix, Note 3) of Carl Hermann. In merohedries the missing symmetry elements along one or

two of the *Blickrichtungen* are symbolized by the identity symbol 1, in order to: (a) distinguish rhombohedral from hexagonal (e.g. 31 from 311); (b) distinguish two different subgroups of the lattice symmetry, when they belong to one and the same point group among the 32 (e.g. 312 and 321, in *h*); optionally, (c) express the complete symbol, for the sake of unity in symbolism, even though no confusion is likely (e.g. 231 in *c*; 611 in *h*; 411 in *t*). In the monoclinic crystal symmetries, there exists only one lattice symmetry direction, which obviously cannot become 1 (since 1 is not a monoclinic merohedry), and no other direction can be written 1 (since it is not a *Blickrichtung* – see Appendix, Note 4).

The derivation (Table 1, column 2) yields 44 different symbols for the 32 traditional point groups (so-called 'crystal classes'). Note that, with the smallest conventional cell determined and used to select the axes of coordinates, it is necessary to consider two subgroups instead of one in four cases in *h* and one case in *t*; examples: 321 and 312, $\bar{4}2m$ and $\bar{4}m2$. Under *o* three different subgroups are possible, $2mm$, $m2m$, $mm2$, instead of the usually given $mm2$; they correspond to actual cases met in practice. It is thus easy to account for the additional twelve crystal-symmetry symbols that bring 32 to 44: 5 for the rhombohedral system, 4 under *h*, 1 under *t*, 2 under *o*. The number of crystal-symmetry symbols in each system appears in the lower right corner of the appropriate pigeonhole (column 2).

The required lattice modes are shown for each lattice symmetry (column 3). In the rhombohedral system *r*, the smallest conventional cell is a primitive rhombohedron rP ; in practice, however, the same rhombohedral lattice is described by means of the hR cell, a triple cell obtained by taking an *h* cell and 'R-centering' it. Note that hR is *not* a mode of the hexagonal lattice: it is an alternate way of defining the rhombohedral lattice (see Appendix, Note 5), whose symmetry is lower than that of the hexagonal lattice hP . The number of possible combinations of crystal symmetries with lattice modes appears in the lower right corner of the pigeon-hole pertaining to the system (column 3). These combinations are the various symbols that can occur for the 73 symmorphic space groups when the edges of the smallest conventional cell are arbitrarily labelled. The numbers of combinations are the same as those of symmetrically distinct symmorphic groups in all systems except *o* (30 vs 13) and *m* (12 vs 6); the total is 96 vs 73. All 96 possibilities are encountered in practice, and the crystallographer must recognize the equivalence of the symbol he has found with that under which the space group is described in *IT*;* special tables (see Appendix, Note 6) for this purpose are provided in *IT* (1952). In a determinative compendium like *Crystal Data* (Donnay & Ondik, 1972–73), where the cell edges are labelled according to a metrical convention ($c < a < b$ in the triclinic case), the larger

number of oriented space-group symbols is definitely an asset.

At one time (*IT*, 1935) setting conventions were based on symmetry instead of metrical considerations. Each crystal symmetry had only one symbol; example: the first subgroup of $6/m2/m2/m$ was written $\bar{6}m2$, whereas the corresponding hemihedry in $4/m2/m2/m$ was written $\bar{4}2m$, in accordance with the mineralogists' tradition. The necessary distinction between the two possibilities was achieved, not by changing the point-group symbol, but by using either a simple or a multiple cell. In *h* if $\bar{6}m2$ had a *P* cell, then our present $\bar{6}2m$ was expressed by $\bar{6}m2$ with an *H* cell (additional nodes at $\frac{1}{3}\frac{2}{3}0$ and $\frac{2}{3}\frac{1}{3}0$). This scheme is given here (columns 4 and 5) for comparison. (Note, however, that the hexagonal and rhombohedral systems appear in the form that is advocated in this paper.) In 1952 the new (English) edition of *IT* went over to the metrical convention in systems *h* and *t*, instead of keeping a conventional way of writing the point-group symbol. In the low symmetry systems *o* and *m*, the 1935 edition of the *Tables* chose to label the cell edges according to symmetry considerations, so that the centering of one face would always be *C*, except in $mm2$, where the twofold axis was chosen as *c* and the one-face-centered lattice was centered either on *C* or on *A* (but never on *B*). In this paper it is recommended to go all the way to metrical conventions and to let the space-group symbol take care of itself. The 1935 scheme correctly enumerates the 73 symmorphic space groups that are possible if the relation between the symmetry and the shape of the cell is ignored.

Finally Table 1 also lists the traditional arrangement of the 32 crystal symmetries (column 6). The five symmetries whose symbols begin with a $\bar{3}$ or a 3 are placed in a special division that is called *rhombohedral* by some and *trigonal* by others (*IT*, 1952). This scheme is inadequate properly to express the crystallographic facts brought out in columns 2 and 3.

The inadequacy of the current classification

The claim to fame of the 32 crystal symmetries is that they purport to represent the result of a mathematical enumeration of the distinct *crystallographic* point groups. Only to the extent that these groups are subject to the limitation imposed on rotation axes by the old Law of Rationality (long before the lattice was even postulated) can one say that they are 'crystallographic'. They pay no heed to the underlying lattice in relation to which the crystal symmetry must be oriented. One can truly state that the 32 symmetries of column 6 are the only possible ones for wooden crystal-form models, which have no lattice to give significance to their various orientations in space. And indeed, if no distinction is made between $3m1$ and $31m$ on the one hand, and $3m$ on the other, so that all three cases merge into one, then it is true that the crystal forms are the same in all three cases. But the interplanar spacings may be dif-

* *Internationale Tabellen zur Bestimmung der Kristallstrukturen* and *International Tables for X-ray Crystallography* are both abbreviated as *IT*, with appropriate dates.

ferent in a given form if it occurs in h or in r , and morphologists can differentiate the two cases on the basis of the law of Bravais (see Appendix, Note 7).

The scheme of 32 crystal symmetries presented as shown (column 6) has many disadvantages. Chief among them is the confusion (see Appendix, Note 8) that results from removing half of the 16 hexagonal groups (column 2) from their lawful supergroup $6/m\ 2/m\ 2/m$ and forcing them into the subgroups of $3\ 2/m$, where they clearly do not belong.

A second disadvantage of column 6 is that it perpetuates the mentality of pre-lattice crystallography: to this day some mineralogy books assign low quartz to the 'rhombohedral' system. 'Rhombohedral', here, is of course a synonym for 'trigonal'. Technically this is not a 'mistake', since quartz does possess point symmetry 32 (in column 6). Albeit, I submit that this information is trivial as compared to the structural and morphological importance of the hexagonal lattice in quartz. Changing 'rhombohedral' into 'trigonal' is not much of an improvement: calcite and quartz are now trigonal; are they both rhombohedral, both hexagonal, and if one is rhombohedral and the other hexagonal, which is the one and which the other?

A third drawback of column 6 stems from the consideration of the lattice complexes in h and r space groups. The two sets are very different. It is well known that all the point positions in which a given lattice complex can be realized occur in 'one and the same crystal system' (Fischer *et al.*, 1973, p. 11, column 1). This statement is true in column 2 and in column 4, but not in column 6, as one can easily check by glancing at the Table of lattice-complex occurrences (Fischer *et al.*, Table 26–27). Hexagonal lattice complexes are realized in crystal symmetries including one 3 as well as in those that include a 6; the rhombohedral complexes materialize only in rhombohedral space groups. Rhombohedral complexes are quite different from hexagonal ones as regards multiplicity. Let us use the triple hR cell and the hP cell, of equal volumes, for ease of comparison. Regardless of the variance of the lattice complexes, multiplicities 1, 2, 4, 8, 24 are found only in hexagonal space groups; 9, 18, 36, only in rhombohedral ones; 3, 6, 12, mostly in hP , except that 3, 6 and 6, 12 also occur in hR , in invariant and univariant complexes, respectively. Of course, two complexes of the same multiplicity will differ markedly in hP and hR .

A fourth drawback of column 6 is met in the study of twins. The classical theory of twinning (see Friedel, 1926) rests on a comparison of the point symmetry of the crystal with that of its lattice (or of one of its superlattices). Examples: in quartz twins the crystal symmetry 321 is a tetartohedry (subgroup of index 4) of the lattice symmetry $6/m\ 2/m\ 2/m$; in calcite twins on $(111)_r = (0001)_h$, the lattice symmetry $3\ 2/m$ is a hemihedry (subgroup of index 2) of the symmetry of the superlattice $6/m\ 2/m\ 2/m$. These apparently subtle, yet fundamental, distinctions are hopelessly obliterated in the scheme of column 6.

Historical perspective

The classification advocated in this paper stems from the recognition of the point symmetry of the lattice as the natural basis for the seven main subdivisions of the crystal kingdom; this approach has been taught since Bravais's time and can be found in many textbooks. Mallard (1879), followed by Friedel (1911, 1926), Donnay & Harker (1940), and Donnay (1942) among others, already stressed the fact that the crystal point symmetries, being the merohedries of the lattice point symmetry, cannot number fewer than 37, inasmuch as the trigonal symmetries (see Appendix, Note 9) must appear as merohedries of two different lattice symmetries. Mallard (1879) also gives a detailed explanation of the necessity for splitting such symbols as $\bar{6}m2$, 32, and $42m$ into $\bar{6}m2 + 62m$, $321 + 312$, $42m + \bar{4}m2$, respectively. The splitting of $mm2$ into $2mm, m2m, mm2$, and the alternate symbols for the one-face centering in the orthorhombic system, have also long been used (see, for instance, Donnay & Harker, 1940; Donnay, Nowacki & Donnay, 1954; Donnay, 1955). In the monoclinic system, two centering modes, C and A , were used by Donnay & Harker (1940) and all three modes, C , A and I , are found in the first edition of *Crystal Data* (Donnay *et al.*, 1954).

The proposal made in this paper, therefore, cannot claim to be original in the usual sense of the word. Whatever originality it has consists in drawing the attention of the crystallographic profession to an anomalous situation that has endured for over a century (see Appendix, Note 10).

Professor Gabrielle Donnay, McGill University, has contributed many discussions and much encouragement. Dr Y. Le Page kindly read the manuscript. Thanks are also due to the two referees for their pertinent suggestions: to sharpen the statement as to which subgroup is the lowest to be included in a given system, 311 in h , in particular; to rewrite, in more detail, the story in Note 8 of the Appendix; to increase the number of references where an original is hard to find.

This work was supported by the Ministère de l'Éducation of the Province of Quebec, under the Programme de formation de chercheurs et d'action concertée.

APPENDIX

Note 1

In the hexagonal system the smallest cell is called 'hexagonal' even though its shape is that of a right prism whose cross section is a 120° rhomb. It has often been said that this cell 'brings out' the symmetry of the lattice. This is, at best, a half-truth: the only polyhedron that performs this feat is Haüy's *forme primitive*, nearly 200 years old and better known nowadays under such

names as Dirichlet domain, Voronoi domain, Brillouin zone, Wigner–Seitz cell, ‘domain of influence’ (*Wirkungsbereich*; see Burzlaff & Zimmermann, 1977, p. 131). In the rhombohedral system the smallest conventional cell is a rhombohedron.

Note 2

The definition of ‘holohedry’ is repeated here to forestall confusion with the bad usage found in *Internationale Tabellen zur Bestimmung der Kristallstrukturen* (1935) [*IT* (1935), p. 274], where $\bar{6}m2$ is called ‘trigonal holohedry’! I traced this mistake back to Schoenflies (1891, p. 229). It is explained, though not justified, by the fact that Schoenflies used rotatory reflection instead of rotatory inversion, so that our $\bar{3}$ was expressed with a $\bar{6}$ ($S_6 = \bar{6}$) and our $\bar{6}$, with a $\bar{3}$ ($D_3 = 3/m$). His ‘trigonal crystal type’ (which he compares to the ‘ordinary crystal system’) contains: $\bar{6}m2$, 32 , $\bar{6}$, $3m$, 3 . His ‘hexagonal crystal type’ includes $\bar{3}2/m$ and $\bar{3}$. Amazing as it may seem, it is clear that, in his mind, ‘holohedry’ was completely divorced from the concept of lattice symmetry. This conclusion is confirmed by the fact that Schoenflies gives eight holohedries (*IT*, 1935, page 33) and gives only one name to each trigonal symmetry; for example, symmetry 3 is called ‘rhombohedral tetartohedry’, but is not recognized as ‘hexagonal ogdohedry’. A similar omission was made in *IT* (1952), Table 3.9.1, where the five trigonal symmetries are considered merohedries of a trigonal (instead of rhombohedral) lattice and no mention is made of the possible hexagonal lattice. This presentation, erroneously attributed to ‘Friedel (1926)’, was corrected by the insertion of an ‘additional note’ in a later printing.

Note 3

The term *Blickrichtungen* is familiar to the former students of C. Hermann in Marburg/Lahn. For its definition, see Burzlaff & Zimmermann (1977, p. 135, sections *f* and *g*). Its first appearance in print could not be ascertained.

Note 4

In the monoclinic system, lattice rows of the type $[u0v]$, which obviously are not *Blickrichtungen*, have abusively been assigned dummy 1’s (*IT*, 1952) to represent coordinate axes, with the resulting destruction of the unique character of the Hermann–Mauguin notation as a *symmetry symbolism* (Mauguin, 1931; Hermann, 1931). Note that such symbols as ‘121’ and ‘112’ would be legitimate in an orthorhombic context, if used to express subgroups of $2/m\ 2/m\ 2/m$, despite the fact that these subgroups are not orthorhombic merohedries. A. L. Patterson, however, has proposed to write them ‘.2.’ and ‘.2.’ [see *IT* (1959), §2.1.12.3 (7), p. 31]; this scheme, which appears to have priority, has been used extensively to symbolize oriented site symmetries in space groups (Fischer, Burzlaff, Hellner & Donnay, 1973; Donnay & Turrell, 1974). The unity

of the Hermann–Mauguin symbolism can only be restored by abandoning the ‘z-unique’ superfluous convention (*cf.* Note 6).

Note 5

Just as the rhombohedral lattice can be described by means of a triple hexagonal cell, it is known that an hexagonal lattice can be defined by a triple rhombohedral cell. This cell, which has its ‘centering’ nodes at $\frac{1}{3}\frac{1}{3}\frac{1}{3}$ and $\frac{2}{3}\frac{2}{3}\frac{2}{3}$ (rhombohedral coordinates), is not used in practice and has received no name. If a name is ever needed, the symbol *rH* is available. The letter *H* is already used to designate a triple hexagonal cell, in which the extra nodes are thirthing the long diagonal of the rhombic mesh; this triple cell *hH* describes the same hexagonal lattice as *hP*. In *rH* it is the vertical body diagonal that is thirded, and *rH* is not a mode of the rhombohedral lattice. The new symbol brings out the parallelism between the two cases: $hR = rP$, $rH = hP$. In symbols *hR* and *rH*, the capital letter tells the crystal system.

Note 6

It is mystifying to observe that Table 6.2.1 in *IT* (1969), p. 545, which gives all the possible symbols for the orthorhombic space groups, offers only three lattice modes in the monoclinic system (four are possible). Although 78 different space-group symbols are displayed on six fully filled columns, they impart only three fourths of the desired information, whereas 29 symbols suffice to present all of it in *Crystal Data* (see, Donnay & Ondik, 1972–1973, p. S-43). The complications arise from the introduction of the so-called ‘first setting’, in addition to the traditional y-unique convention (dubbed ‘second’ setting!). This permutation of coordinate axes is of questionable or marginal usefulness; it clutters the *Tables*; it makes for confusion; it violates the spirit of the Hermann–Mauguin symbolism (*cf.* Note 4) and will entail duplication in the description of the monoclinic space groups, which already needs considerable lengthening to cope with difficulties that are due to Nature’s own doing and could well do without additional man-made complications. Crystallographers must face the facts that, once the smallest conventional cell is chosen and labelled, the lattice mode will be expressed by one of four different symbols and the plane of symmetry, in the space-group symbol, by one of four different letters. It would appear to be of much less importance to be given the option of calling the axes *zxy* instead of *yzx*, for this is not a matter of ‘setting’: the crystal is set the same way with respect to its lattice in either case; the only real difference is one of algebraic semantics. Many thorny problems will vanish when the ‘z-unique’ option finally disappears from *IT* after a 25-year probation period. It can still remain open, under the terms of the Stockholm decision, to the virtuosi of symmetry theory, who in any event do not need any help from *IT* to practice their craft.

Note 7

The French crystallographer Henri Ungemach (1877–1936) still used a nomenclature of crystal forms in which rhombohedral hemihedral and hexagonal tetartohedral forms, for instance, were given different names (despite their being geometrically the same); these names were intended to stress differences stemming from the lattice involved in each case. Example: the trigonal trapezohedron in low quartz was called a tetartodipyramid; had the quartz lattice been rhombohedral instead of hexagonal, it would have been a hemiscalenohedron. (These names have long been abandoned, but they are typical of an era when crystallographers determined morphological lattices.)

Note 8

Let me give an example of this confusion. A few years ago I received a letter from a practicing crystallographic consultant, who actually thought that 'trigonal *P*' and 'trigonal *R*' were two modes of a 'trigonal lattice', just as, in the cubic system, the lattice modes are 'cubic *P*', 'cubic *I*' and 'cubic *F*'. He had observed that space groups such as *P*3 and *R*3 appear in *IT* on pages marked 'trigonal' in the same manner as *P*23, *I*23, *F*23 are found under 'cubic'. He was telling me that he knew that 'all hexagonal space groups' have the 'hexagonal *P*' lattice; he also knew that 'trigonal *R*' meant 'rhombohedral lattice', but he could not anywhere find any description of the 'trigonal *P* lattice'! (I do not blame him for taking the *Tables* too literally: I might have been puzzled myself, had I not served so many years on the IUCr Commission on *International Tables*.) On learning that 'trigonal *P*' and 'hexagonal *P*' designate the same lattice, my correspondent was so shocked that he wanted everyone else to know and insisted I should publish a clarifying note about it!

Note 9

'Trigonal symmetries' is a convenient term that can be retained, when the lattice is not considered, to designate the five point groups that contain a single threefold axis, with or without the center ($\bar{1}$), but without a mirror (*m*) perpendicular to it ($3/m = \bar{6}$).

Note 10

This paper is offered in the hope that practicing

crystallographers will make their voices heard, through their National Committees, to obtain the needed revisions of Vol. I of *IT*.

References

- BURZLAFF, H. & ZIMMERMANN, H. (1977). *Symmetriehlehre*. (Vol. 1 of *Kristallographie: Grundlagen und Anwendung* series, Edited by H. BURZLAFF & G. THIELE). Stuttgart: Thieme. (See pp. 134–135.)
- DONNAY, J. D. H. (1942). *Univ. Toronto Studies, Geol. Series*, No. 47, 33–51. (See Table II.)
- DONNAY, J. D. H. (1955). *Crystal Morphology*. Article in *Encycl. Amer.* pp. 277–277d. (See Table, p. 277b.)
- DONNAY, J. D. H. & HARKER, D. (1940). *Nat. Canad.* **67**, 33–69; 160. (See *Tables of space-group omissions*.)
- DONNAY, J. D. H., NOWACKI, W. & DONNAY, G. (1954). *Crystal Data*. 1st ed., *Geol. Soc. Amer., Mem.* 60. (See Appendix C, pp. 585–596.)
- DONNAY, J. D. H. & ONDIK, H. M. (1972–73). (General Editors) *Crystal Data*, 3rd ed. Vol. I. *Organic* (1972). Vol. II. *Inorganic* (1973). Washington: NSRDS, Bureau of Standards and Joint Committee on Powder Diffraction Standards (See Supplement II in either volume.)
- DONNAY, J. D. H. & TURRELL, G. (1974). *Chem. Phys.* **6**, 1–18. (See *Tables of oriented symmetries in space groups*.)
- FISCHER, W., BURZLAFF, H., HELLNER, E. & DONNAY, J. D. H. (1973). *Space Groups and Lattice Complexes. Natl. Bur. Stand. Mon.* 134. Washington, DC: US Government Printing Office. (See Tables 33–34, pp. 138–148.)
- FRIEDEL, G. (1911). *Leçons de Cristallographie*. Paris: Herman. (See *Les 32 types de symétrie compatibles avec la loi d'Häuy*.)
- FRIEDEL, G. (1926). *Leçons de Cristallographie*. Paris: Berger-Levrault: Reprinted (1974), Paris: Blanchard. (See pp. 58–60.)
- HERMANN, C. (1931). *Z. Kristallogr.* **76**, 559–561. *Internationale Tabellen zur Bestimmung der Kristallstrukturen* (1935). Vol. I. Berlin: Borntraeger.
- International Tables for X-ray Crystallography* (1952). Vol. I. (2nd ed., 1965; 3rd ed., 1969). Birmingham: Kynoch Press.
- International Tables for X-ray Crystallography* (1959). Vol. II. Birmingham: Kynoch Press.
- MALLARD, E. (1879). *Traité de Cristallographie*, Vol. I. Paris: Dunod.
- MAUGUIN, C. (1931). *Z. Kristallogr.* **76**, 542–558.
- SCHOENFLIES, A. (1891). *Krystallsysteme und Krystallstructur*. Leipzig: Teubner.